

# Closed superstrings in magnetic field: instabilities and supersymmetry breaking

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## Abstract

We consider a 2-parameter class of solvable closed superstring models which ‘interpolate’ between Kaluza-Klein and dilatonic Melvin magnetic flux tube backgrounds. The spectrum of string states has similarities with Landau spectrum for a charged particle in a uniform magnetic field. The presence of spin-dependent ‘gyromagnetic’ interaction implies breaking of supersymmetry and possible existence (for certain values of magnetic parameters) of tachyonic instabilities. We study in detail the simplest example of the Kaluza-Klein Melvin model describing a superstring moving in flat but non-trivial 10-d space containing a 3-d factor which is a ‘twisted’ product of a 2-plane and an internal circle. We also discuss the compact version of this model constructed by ‘twisting’ the product of the two groups in  $SU(2) \times U(1)$  WZNW theory without changing the local geometry (and thus the central charge). We explain how the supersymmetry is broken by continuous ‘magnetic’ twist parameters and comment on possible implications for internal space compactification models.

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## 1. Introduction

An important problem in string theory is to study how quantised strings propagate in non-trivial backgrounds described by conformal 2d models. This may help to understand the structure of the space of exact string solutions as well as certain generic properties of string theory like the existence ‘critical’ or ‘maximal’ values of fields, instabilities, mechanisms of supersymmetry breaking, residual symmetries of string spectrum, etc.

Magnetic field is one of the simplest probes of the spectrum and critical properties of a physical system. In the context of a gravitational theory like closed string theory, a magnetic background is accompanied by a curvature of space-time. In spite of that, certain closed string magnetic models can be solved exactly. In particular, these are static flux tube type configurations with approximately uniform magnetic field which generalize the Melvin solution of the Einstein-Maxwell theory [1]. Such backgrounds are exact solutions of (super)string theory [2,3] and, moreover, the spectrum of the corresponding unitary conformal string models can be explicitly determined [3,4].

Below we shall first review the solution [4] of this class of magnetic models. In the last section we shall discuss related compact models which may be used for string compactifications and the issue of supersymmetry breaking induced by ‘magnetic’ twist parameters.

Before turning to closed string theory let us first recall the solution of similar uniform magnetic field problems in particle (field) theory and open string theory.

### 1.1. Point particles and open strings in magnetic field

The reason why the quantum-mechanical or field-theoretical problem of a particle in a uniform abelian (electro)magnetic field is exactly solvable is that the action  $I = \int d\tau [\dot{x}^\mu \dot{x}^\mu + ie\dot{x}^\mu A_\mu(x)]$  (which determines the Hamiltonian in quantum mechanics and the heat kernel in field theory) becomes gaussian if the field strength is constant,  $A_\mu = -\frac{1}{2}F_{\mu\nu}x^\nu$ ,  $F_{\mu\nu} = \text{const}$ . Assuming the magnetic field is directed along  $x^3$ -axis (so that  $F_{ij} = H\epsilon_{ij}$ ,  $i, j = 1, 2$ ) and introducing  $x = x_1 + ix_2$ ,  $a = -i(\partial_x^* + \frac{1}{4}eHx)$ ,  $a^\dagger = -i(\partial_x - \frac{1}{4}eHx^*)$ ,  $[a, a^\dagger] = \frac{1}{2}eH$ , one can reduce the corresponding quantum-mechanical problem to a free oscillator one. The resulting energy spectrum is the special ( $S = 0$ ) case of the Landau spectrum for a particle of charge  $e$ , mass  $M_0$  and third component of the spin  $S$  (we assume  $eH > 0$ )

$$E^2 = M_0^2 + p_3^2 - 2eHJ, \quad J \equiv -l - \frac{1}{2} + \frac{1}{2}gS, \quad l = 0, 1, 2, \dots \quad (1.1)$$

Here  $l$  is the Landau level number (which replaces the continuous momenta  $p_1, p_2$ ) and  $g$  is the gyromagnetic ratio which is  $1/S$  for minimally coupled particles but can be equal to 2 for non-minimally coupled ones. Thus  $E^2$  can become negative for large enough values of  $H$ , e.g.,  $H > H_{crit} = M_0^2/e$  for spin 1 charged states. That applies, for example, to  $W$ -bosons in the context of electroweak theory [5] suggesting the presence of a transition

to a phase with a  $W$ -condensate. In the case of unbroken gauge theory with massless charged vector particles the instability is present for any (e.g., infinitesimal) value of the magnetic field [5]. This infra-red instability of a magnetic background is not cured by supersymmetry, i.e. it remains also in supersymmetric gauge theories (e.g., in ultra-violet finite  $N = 4$  supersymmetric Yang-Mills theory) since the small fluctuation operator for the gauge field  $-\delta_{\mu\nu}D^2 - 2F_{\mu\nu}$  still has negative modes due to the ‘anomalous magnetic moment’ term. This is not surprising given that the magnetic spin-dependent coupling breaks supersymmetry.

The meaning of this instability is that the expansion is carried out near a classical solution of the Yang-Mills equations (abelian  $A_i = -\frac{1}{2}H\epsilon_{ij}x^j$  belonging to the Cartan subalgebra) which is *not* a vacuum one: the energy is proportional to  $H^2$  and thus is minimal for  $H = 0$ . As a result, a non-zero magnetic field will tend to dissipate. The presence of tachyonic modes in the magnetic background with infinitesimal (but non-vanishing  $H$ ) does not indicate an instability of the trivial vacuum but only that of a configuration with  $H \neq 0$  (this corresponds to a resummation of the expansion in small perturbations near the trivial vacuum).

The solution of the uniform magnetic field problem in the open string theory is similar to the one in the particle case. Indeed, the open string is coupled to the (abelian) vector field only at its ends,  $I = \frac{1}{4\pi\alpha'} \int d^2\sigma \partial_a x^\mu \partial^a x^\mu + ie \int d\tau A_\mu(x) \dot{x}^\mu$ , and thus for the abelian background  $A_\mu = -\frac{1}{2}F_{\mu\nu}x^\nu$  the resulting gaussian path integral can be computed exactly [6]. This is a consistent ‘on-shell’ problem since  $F_{\mu\nu} = \text{const}$  is an exact solution of the effective field equations of the open string theory. The corresponding 2d world-sheet theory is a conformal field theory [7] which can be solved explicitly in terms of free oscillators (thus representing a string generalization of the Landau problem in quantum particle mechanics). As a result, one is able to determine the spectrum of an open string moving in a constant magnetic field [7,8,9]. The Hamiltonian ( $L_0$ ) of the open (super)string in a constant magnetic field is given by [7,9]

$$\hat{\mathcal{H}} = \frac{1}{2}\alpha'(-E^2 + p_\alpha^2) + \hat{N} - \gamma\hat{J} , \quad (1.2)$$

$$\hat{\gamma} \equiv \frac{2}{\pi} \left| \arctan(\frac{1}{2}\alpha'\pi e_1 H) + \arctan(\frac{1}{2}\alpha'\pi e_2 H) \right| , \quad 0 \leq \hat{\gamma} < 1 .$$

Here  $e_1, e_2$  are charges at the two ends of the string and  $H$  is the magnetic field,  $F_{ij} = H\epsilon_{ij}$ .  $\hat{N} = 0, 1, 2, \dots$  is the number of states operator and  $\hat{J} = -l - \frac{1}{2} + S$  is the angular momentum operator of the open string in the (1,2) plane. The energy spectrum is found from the constraint  $\hat{\mathcal{H}} = 0$  and for small  $e_i H$  is in agreement with (1.1) with  $g = 2$ . A novel feature of this spectrum as compared to the free (super)string one is the presence of tachyonic states above certain critical values of the magnetic field ( $H_{crit} \sim \alpha'^{-1}\hat{N}/S \sim \alpha'^{-1}$ ). Thus the constant magnetic field background is unstable in the open string theory (as it is in the non-abelian gauge theory). A qualitative reason for this instability is that the free open string spectrum contains electrically charged higher spin massive particle states and the latter have (approximately) the Landau spectrum (1.1) ( $\hat{\gamma} \approx \alpha'(e_1 + e_2)H$  for a weak field). The tachyonic states appear only in the bosonic (Neveu-Schwarz) sector and only on the leading Regge trajectory ( $l = 0$ , maximal  $S$  for a given mass level  $\hat{N}$ ,  $S = \hat{N} + 1$ ).

## 1.2. Closed strings and Melvin-type magnetic flux tube backgrounds

Similar background magnetic field problem is also exactly solvable in *closed* (super)string theories [10,3,4]. This may be unexpected since the abelian vector field is now coupled to the internal points of the string, and such interaction terms, e.g.,  $\Delta L = \partial y \bar{\partial} y + A_\mu(x) \partial x^\mu \bar{\partial} y + \tilde{A}_\mu(x) \bar{\partial} x^\mu \partial y + \dots$ , do not become gaussian for  $A_\mu = -\frac{1}{2} F_{\mu\nu} x^\nu$ . Here  $y \in (0, 2\pi R)$  is a compact internal Kaluza-Klein coordinate that ‘charges’ the string. The spectrum of the closed string compactified on a circle contains states with arbitrarily large masses, spins *and* charges  $Q_{L,R} = mR^{-1} \pm w\alpha'^{-1}R$ , ( $m, w = 0, \pm 1, \dots$ ). One difference compared to the open string case is that there are infinitely many possible values of charges. Another is that both  $Q_L$  and  $Q_R$  are, in general, non-vanishing and coupled to (a combination of) two abelian vector fields ( $G_{5\mu}$  and  $B_{5\mu}$ ).<sup>1</sup>

The important observation is that in contrast to the tree level abelian open string case, the  $F_{\mu\nu} = \text{const}$  background in flat space does not represent a solution of a *closed* string theory, i.e. the above interaction terms added to the free string Lagrangians do not lead to conformally invariant 2d  $\sigma$ -models. Since the closed string theory contains gravity, a uniform magnetic field, which has a non-vanishing energy, must curve the space (as well as possibly induce other ‘massless’ background fields). One should thus first find a consistent conformal model which is a closed string analogue of the uniform magnetic field background in the flat space field (or open string) theory and then address the question of its solvability. Remarkably, it turns out that extra terms which should be added to the closed string action in order to satisfy the conformal invariance condition (i.e. to satisfy the closed string effective field equations) produce solvable 2d models. Like the particle and open string models, these models are exactly solvable in terms of free oscillators (one is able to find explicitly the spectrum, partition function, etc).

A simple example of an approximately uniform magnetic field background in the Einstein-Maxwell theory is the static cylindrically symmetric Melvin ‘magnetic universe’ or ‘magnetic flux tube’ solution [1]. It has  $R^4$  topology and can be considered [12] as a gravitational analogue of the abelian Higgs model vortex [13] with the magnetic pressure (due to repulsion of Faraday’s flux lines) being balanced not by the Higgs field but by the gravitational attraction. The magnetic field is approximately constant inside the tube and decays to zero at infinity in the direction orthogonal to  $x_3$ -axis. The space is not, however, asymptotically euclidean: the  $(\rho, \varphi)$  2-plane orthogonal to  $x_3$  asymptotically closes at large  $\rho$  (so the solution should be interpreted not as a flux tube embedded in an approximately flat space, but rather as a ‘magnetic universe’). Several interesting features of the Melvin solution in the context of Kaluza-Klein (super)gravity (e.g., instability against monopole or magnetic black hole pair creation) were discussed in [12,14,15].

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<sup>1</sup> As a result, the gyromagnetic ratio of closed string states is  $g \leq 2$ , e.g.,  $g = 1$  for standard non-winding Kaluza-Klein states [11,3].

This Einstein-Maxwell (' $a = 0$ ') solution has two direct analogues among solutions of the low-energy closed string theory (superstring toroidally compactified to  $D = 4$ ). Assuming  $x^5 = y$  is a compact internal coordinate, the  $D = 5$  string effective action can be expressed in terms of  $D = 4$  fields: metric  $G_{\mu\nu}$ , dilaton  $\phi$ , antisymmetric tensor  $B_{\mu\nu}$ , two vector fields  $\mathcal{A}_\mu$  and  $\mathcal{B}_\mu$  (related to  $G_{5\mu}$  and  $B_{5\mu}$ ) and the 'modulus'  $\sigma$ . The dilatonic (' $a = 1$ ') and Kaluza-Klein (' $a = \sqrt{3}$ ') Melvin solutions have zero  $B_{\mu\nu}$  but  $\phi$  or  $\sigma$  being non-constant. Starting with the  $D = 5$  bosonic string effective action and assuming that one spatial dimension  $x^5$  is compactified on a small circle, one finds (ignoring massive Kaluza-Klein modes) the following dimensionally reduced  $D = 4$  action

$$S_4 = \int d^4x \sqrt{G} e^{-2\Phi} [ R + 4(\partial_\mu \Phi)^2 - (\partial_\mu \sigma)^2 \quad (1.3)$$

$$- \frac{1}{12}(\hat{H}_{\mu\nu\lambda})^2 - \frac{1}{4}e^{2\sigma}(F_{\mu\nu}(\mathcal{A}))^2 - \frac{1}{4}e^{-2\sigma}(F_{\mu\nu}(\mathcal{B}))^2 + O(\alpha') ] ,$$

where  $\hat{G}_{\mu\nu} = G_{\mu\nu} - G_{55}\mathcal{A}_\mu\mathcal{A}_\nu$ ,  $F_{\mu\nu}(\mathcal{A}) = 2\partial_{[\mu}\mathcal{A}_{\nu]}$ ,  $F_{\mu\nu}(\mathcal{B}) = 2\partial_{[\mu}\mathcal{B}_{\nu]}$ ,  $\hat{H}_{\lambda\mu\nu} = 3\partial_{[\lambda}B_{\mu\nu]} - 3\mathcal{A}_{[\lambda}F_{\mu\nu]}(\mathcal{B})$ ,  $\Phi = \phi - \frac{1}{2}\sigma$ . The effective equations following from (1.3) have a 3-parameter  $(\alpha, \beta, q)$  class of stationary axisymmetric (electro)magnetic flux tube solutions [3]. The most interesting subclass of these solutions ( $\alpha = \beta$ ) describes *static* magnetic flux tube backgrounds ( $\beta$  and  $q$  are magnetic field strength parameters). It contains the  $a = \sqrt{3}$  and  $a = 1$  Melvin solutions [16] as the special cases of  $\beta = 0$  and  $\beta = q$ . The four-dimensional geometry is given by (in terms of the string metric in (1.3);  $x^\mu = (t, \rho, \varphi, x^3)$ )

$$ds_4^2 = -dt^2 + d\rho^2 + F(\rho)\tilde{F}(\rho)\rho^2 d\varphi^2 + dx_3^2 , \quad (1.4)$$

$$\mathcal{A}_\varphi = qF(\rho)\rho^2 , \quad \mathcal{B}_\varphi = -\beta\tilde{F}(\rho)\rho^2 , \quad (1.5)$$

$$e^{2(\phi-\phi_0)} = \tilde{F}(\rho) , \quad e^{2\sigma} = \tilde{F}(\rho)F^{-1}(\rho) , \quad F \equiv \frac{1}{1+q^2\rho^2} , \quad \tilde{F} \equiv \frac{1}{1+\beta^2\rho^2} .$$

There are two magnetic fields with the effective strengths  $q$  and  $\beta$  (when both are non-vanishing the radius of  $\varphi$ -circle goes to zero at large  $\rho$ ). In particular,<sup>2</sup> the ' $a = \sqrt{3}$  Melvin' background is ( $\beta = 0$ ,  $q \neq 0$ ):

$$ds_4^2 = -dt^2 + d\rho^2 + F(\rho)\rho^2 d\varphi^2 + dx_3^2 , \quad (1.6)$$

$$\mathcal{A} = qF(\rho)\rho^2 d\varphi , \quad e^{2\sigma} = F^{-1} = 1 + q^2\rho^2 , \quad \mathcal{B} = B = 0 , \quad \phi = \phi_0 ,$$

and the ' $a = 1$  Melvin' background is ( $\beta = q \neq 0$ ):

$$ds_4^2 = -dt^2 + d\rho^2 + \tilde{F}^2(\rho)\rho^2 d\varphi^2 + dx_3^2 , \quad (1.7)$$

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<sup>2</sup> In these special cases there is effectively just one non-trivial vector and one scalar so that the Einstein-frame action can be put into the form  $\int d^4x \sqrt{G'} [R' - \frac{1}{2}(\partial_\mu \psi)^2 - \frac{1}{4}e^{-\alpha\psi} F_{\mu\nu}^2]$ .

$$\mathcal{A} = -\mathcal{B} = \beta \tilde{F}(\rho) \rho^2 d\varphi, \quad e^{2(\phi-\phi_0)} = \tilde{F} = (1 + \beta^2 \rho^2)^{-1}, \quad B = \sigma = 0.$$

The string model corresponding to the  $a = 1$  Melvin background [16] was constructed in [2] and solved in [3]. The general case of arbitrary  $(\beta, q)$  was studied in [3]. The model describing  $a = \sqrt{3}$  Melvin background is the simplest possible special case. The reason is that when viewed from higher dimensions the background (1.6) corresponds to a *flat* (but globally non-trivial) 5-dimensional space-time [14,15].<sup>3</sup> This is why the associated string model is explicitly solvable.

The remarkable simplicity of the  $a = \sqrt{3}$  Melvin string model makes it a good pedagogical example which we shall discuss first (Section 2), before turning to more general  $(\beta, q)$  models (which no longer correspond to flat higher dimensional spaces but are still solvable). The quantum Hamiltonian of the corresponding type II superstring model will be equal to the free superstring one plus terms linear and quadratic in angular momentum operators. As a result, the mass spectrum can be explicitly determined. We will show that supersymmetry is broken and that there exist intervals of values of moduli parameters (Kaluza-Klein radius and magnetic field strength) for which the model is unstable. The string partition function on the torus is IR finite or infinite depending on the values of the parameters. We shall consider both the Ramond-Neveu-Schwarz and the light-cone Green-Schwarz formulation of the theory (in the latter the breaking of supersymmetry is related to the absence of Killing spinors in the Melvin background).

In Section 3 the results obtained for the  $a = \sqrt{3}$  Melvin model will be generalized to the  $(\beta, q)$  class of static magnetic flux tube models and, in particular, to the  $a = 1$  Melvin model. We shall explain the reason for solvability of these models and clarify the nature of perturbative instabilities that appear for generic values of the magnetic field parameters.

In Section 4 we shall first explain the relation between the  $a = \sqrt{3}$  Melvin model and superstring compactifications on twisted tori where supersymmetry is broken by discrete rotation angles. Then we shall consider the compact version of the  $a = \sqrt{3}$  Melvin model which is obtained by ‘twisting’  $SU(2) \times U(1)$  WZNW model and discuss the issue of supersymmetry breaking by the twist parameters.

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<sup>3</sup> This is an interesting example of a Kaluza-Klein background which looks non-trivial (curved) from lower-dimensional point of view but is actually flat as 5-dimensional space. That means that the total contribution of the three 4-dimensional fields (metric, vector and scalar) in any local observable will vanish. However, the global properties of the corresponding string theory will be non-trivial. Note also that since the effective Kaluza-Klein radius  $e^\sigma R$  grows with  $\rho$ , the Kaluza-Klein interpretation may not apply for large  $q$  (see [15]).

## 2. Superstring model for $a = \sqrt{3}$ Melvin background

### 2.1. Bosonic string model

Let us consider the closed bosonic string propagating in the space  $M^D = M^5 \times T^{D-5}$ , where  $T^n$  is a torus and  $M^5 = R_t \times R_{x_3} \times \mathcal{M}^3$ .  $\mathcal{M}^3$  is flat but globally non-euclidean space which can be represented as a twisted product (symbolically,  $\mathcal{M}^3 = R^2 * S^1$ ) of the 2-plane  $R^2(\rho, \varphi)$  with Kaluza-Klein circle  $S^1$  ( $y \in (0, 2\pi R)$ ) (or as a bundle with  $S^1$  as a base and  $R^2$  as a fibre). It can also be obtained [15] by factorizing  $R^3$  over the group generated by translations in two angular directions: in the coordinates where  $ds^2 = d\rho^2 + \rho^2 d\theta^2 + dy^2$  one should identify the points  $(\rho, \theta, y) = (\rho, \theta + 2\pi n + 2\pi q R m, y + 2\pi R m)$  ( $n, m = \text{integers}$ ), i.e. combine the shift by  $2\pi R$  in  $y$  with a rotation by an arbitrary angle  $2\pi q R$  in the 2-plane. In terms of the globally defined  $2\pi$ -periodic coordinate  $\varphi$  the metric of  $\mathcal{M}^3$  is

$$ds^2 = d\rho^2 + \rho^2(d\varphi + qdy)^2 + dy^2 . \quad (2.1)$$

It is flat since locally one may introduce the coordinate  $\theta = \varphi + qy$  and decouple  $y$  from  $\rho, \varphi$ . The global structure of this 3-space is non-trivial: the fixed  $\rho$  section is a 2-torus (with  $\rho$ -dependent conformal factor and complex modulus) which degenerates into a circle at  $\rho = 0$  (the space is actually regular everywhere, including  $\rho = 0$ ).

The Lagrangian describing string propagation in such flat but globally non-trivial  $M^D$  is

$$L = L_0 + L_1 , \quad L_0 = -\partial_a t \partial^a t + \partial_a x_\alpha \partial^a x^\alpha , \quad (2.2)$$

$$L_1(\mathcal{M}^3) = \partial_a \rho \partial^a \rho + \rho^2 (\partial_a \varphi + q \partial_a y) (\partial^a \varphi + q \partial^a y) + \partial_a y \partial^a y . \quad (2.3)$$

Here  $\rho \geq 0$  and  $0 < \varphi \leq 2\pi$  correspond to the cylindrical coordinates on a  $(x_1, x_2)$ -plane,  $y$  is a ‘Kaluza-Klein’ coordinate with period  $2\pi R$ , and  $x^\alpha$  include the flat  $x^3$ -coordinate of  $D = 4$  space-time and, e.g., 21 (or 5 in the superstring case) internal coordinates compactified on a torus. To give the 4-dimensional interpretation to this model  $L_1$  should be represented in the ‘Kaluza-Klein’ form

$$L_1 = \partial_a \rho \partial^a \rho + F(\rho) \rho^2 \partial_a \varphi \partial^a \varphi + e^{2\sigma} (\partial_a y + \mathcal{A}_\varphi \partial_a \varphi) (\partial^a y + \mathcal{A}_\varphi \partial^a \varphi) , \quad (2.4)$$

where  $F^{-1} = e^{2\sigma} = 1 + q^2 \rho^2$ ,  $\mathcal{A}_\varphi = q \rho^2 F$ . The resulting  $D = 4$  background (metric, Abelian vector field  $\mathcal{A}_\mu$  and scalar  $\sigma$ ) is indeed the  $a = \sqrt{3}$  Melvin geometry (1.6). The parameter  $q$  thus has the interpretation of the magnetic field strength at the core of the

flux tube. From the 4-dimensional point of view this model describes the motion of charged string states in the  $a = \sqrt{3}$  Melvin magnetic flux tube background.<sup>4</sup>

Although the 5-space is flat, the string theory (2.3) will be non-trivial already at the classical level (due to the existence of winding string states) and also at the quantum level in the non-winding sector (where there will be a ‘magnetic’ coupling to the total angular momentum in the 2-plane). This represents an example of a gravitational 5d (space-time) Aharonov-Bohm-type phenomenon: the value of the magnetic field strength parameter  $q$  does not influence the (zero) curvature of the space but affects the global properties like masses of string states.

Since  $\mathcal{M}^3$  is flat, the model (2.3) is conformal for arbitrary values of the two parameters  $q$  and  $R$ . Certain values of these moduli are special: if  $qR = n$ ,  $n = 0, \pm 1, \dots$ , the coordinate  $\theta$  is globally defined ( $2\pi$  periodic) and so (2.4) is equivalent to a free bosonic string theory compactified on a circle.<sup>5</sup> Models with  $n < qR < n + 1$  are equivalent to models with  $0 < qR < 1$ . This periodicity condition in  $qR$  will be modified in the superstring theory: because of the presence of fermions of half-integer spin  $n$  will be replaced by  $2n$ , i.e. only models with  $qR = 2n$  will be trivial (more generally, superstring theories with  $(R, q)$  and  $(R, q + 2nR^{-1})$  will be equivalent).

The Lagrangian (2.4) has the following useful form ( $x = x_1 + ix_2 = \rho e^{i\varphi}$ ) :

$$\begin{aligned} L_1 &= (\partial_a x_i - q\epsilon_{ij}x_j\partial_a y)(\partial^a x_i - q\epsilon_{ij}x_j\partial^a y) + \partial_a y\partial^a y \\ &= D^a x D_a^* x^* + \partial_a y\partial^a y, \quad D_a \equiv \partial_a + iA_a, \quad A_a \equiv q\partial_a y, \end{aligned} \tag{2.5}$$

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<sup>4</sup> We assume that  $q$  is a continuous parameter. There is no reason for its quantization at the level of string model (2.3). Given the magnetic flux tube (1.6) interpretation of this model, one may, however, wonder how this is reconciled with the flux quantization in similar magnetic backgrounds like the Higgs scalar vortex or magnetic monopole on 2-sphere. Though the magnetic flux through the  $(\rho, \varphi)$  2-plane is finite in the case of the Melvin background, the topological argument for its quantization (cf. [12]) does not apply since the 2-space is non-compact (the  $\rho = \infty$  point is not part of the space). Given a magnetic field configuration with a finite flux through a 2-plane, the flux may be quantised once charges are added since this may corresponds to a state of minimal energy (cf. the case of the Cooper pairs in superconductors; the minimal energy condition leads also to the asymptotic condition  $D_\mu \phi = 0$ ,  $ieA_\mu = \partial_\mu \phi$ , etc., in the case of the scalar vortex).

<sup>5</sup> The trivial models with  $qR = n$  may still look non-trivial from lower dimensional Kaluza-Klein point of view. The equivalence between higher dimensional and lower dimensional descriptions is, of course, established once the contribution of the whole tower of higher massive Kaluza-Klein states is taken into account.



where the 2d gauge potential  $A_a$  is flat (locally pure gauge). Since  $y$  is compact, the effect of this gauge potential will be non-trivial if the world sheet will have non-trivial holonomy (2d or ‘world sheet’ Aharonov-Bohm effect).

The flatness of the potential  $A_a = q\partial_a y$  in (2.5) implies that  $x$  can be formally ‘rotated’ to decouple it from  $y$ . Then  $y$  satisfies the free-field equation and  $x$  is also expressed in terms of free fields. The only interaction which effectively survives in the final expressions is the coupling of  $x$  to the derivative of the zero mode part of  $y$  (e.g.,  $y_* = y_0 + 2\alpha' p\tau + 2Rw\sigma$  in the case of the cylinder as a world sheet). It is then straightforward to carry out the canonical quantization procedure, expressing all observables in terms of free oscillators. The resulting Hamiltonian will be given by the sum of the free string Hamiltonian plus  $O(q)$  and  $O(q^2)$  terms depending on the left and right components of the free string angular momentum operators  $\hat{J}_L$  and  $\hat{J}_R$  [3,4].

This bosonic string model is stable in the non-winding sector, where there are no new instabilities in addition to the usual flat space tachyon [3]. This means, in particular, that the Kaluza-Klein field theory corresponding to the Melvin background is perturbatively stable with respect to the ‘massless’ (graviton, vector, scalar) *and* massive perturbations (the theory may still be unstable at a non-perturbative level [15]). At the same time, there exists a range of parameters  $q$  and  $R$  for which there are tachyonic states in the *winding* sector. This instability (whose origin is essentially in the gyromagnetic coupling term  $wqR(\hat{J}_R - \hat{J}_L)$  which may have a negative sign, see below and cf. (1.1),(1.2)) is not related to the presence of the flat bosonic string tachyon and survives also in the superstring case [4].

## 2.2. Solution of the superstring model

Let us now consider the type II superstring version of (2.2) using first the RNS formulation of the model. The (1,1) world-sheet supersymmetric extension of the model (2.3),(2.5) has the form ( $x^\mu \equiv (x^i, y)$ )

$$L_{\text{RNS}} = G_{\mu\nu}(x)\partial_+ x^\mu \partial_- x^\nu + \lambda_{Rm} D_+ \lambda_R^m + \lambda_{Lm} D_- \lambda_L^m, \quad (2.6)$$

$$D_{\pm n\mu}^m \equiv \delta_n^m \partial_\pm + \omega_{n\mu}^m \partial_\pm x^\mu.$$

$\lambda^m = e_\mu^m \lambda^\mu$  are vierbein components of the 2d Majorana-Weyl spinors and  $\omega_{n\mu}^m$  is flat spin connection (so that the quartic fermionic terms are absent).<sup>6</sup> In terms of the left and right Weyl spinors  $\lambda = \lambda_1 + i\lambda_2$  corresponding to  $x = x_1 + ix_2$  and  $\lambda^y \equiv \psi$ , we get (cf. (2.5))

$$L_{\text{RNS}} = \frac{1}{2}(D_+ x D_-^* x^* + c.c.) + \partial_+ y \partial_- y + \lambda_R^* D_+ \lambda_R + \lambda_L^* D_- \lambda_L \quad (2.7)$$

---

<sup>6</sup> In the basis  $e^i = dx^i - q\epsilon^{ij}x_j dy$ ,  $e^y = dy$ , the spin connection 1-form has the following components:  $\omega^{ij} = -q\epsilon^{ij}dy$ ,  $\omega^{iy} = 0$ .

$$+ \psi_R \partial_+ \psi_R + \psi_L \partial_- \psi_L , \quad D_\pm \equiv \partial_\pm + iq \partial_\pm y ,$$

where the covariant derivative  $D_\pm$  is the same as in (2.5), i.e. it contains the flat 2d  $U(1)$  potential. This means that, as in the bosonic case, it is possible to redefine the fields  $x, \lambda$  so that the only non-trivial coupling will be to the zero mode of  $y$ . Although it may seem that, as in the bosonic case, the model with  $qR = n$  should be equivalent to the free superstring theory compactified on a circle (since for  $qR = n$  one can, in principle, eliminate the coupling terms in (2.7) by rotating the fields) this will not actually be true unless the integer  $n$  is even,  $n = 2k$ . The non-triviality for  $n = 2k + 1$  is directly related to the presence of space-time fermions in the spectrum, which change sign under  $2\pi$  spatial rotation accompanying the periodic shift in  $y$ . This will be obvious in the GS formulation (see below).

Taking the world sheet to be a cylinder ( $0 < \tau < \infty, 0 < \sigma \leq \pi$ ) so that  $x, y, \lambda$  obey the usual closed-string boundary conditions

$$x(\tau, \sigma + \pi) = x(\tau, \sigma) , \quad y(\tau, \sigma + \pi) = y(\tau, \sigma) + 2\pi R w , \quad w = 0, \pm 1, \dots , \quad (2.8)$$

$$\lambda_{R,L}(\tau, \sigma + \pi) = \pm \lambda_{R,L}(\tau, \sigma) , \quad (2.9)$$

we can solve the classical equations corresponding to (2.7) by introducing the fields  $X$  and  $\Lambda_{R,L}$ , which satisfy the free string equations but have ‘twisted’ boundary conditions ( $\sigma_\pm \equiv \tau \pm \sigma$ )<sup>7</sup>

$$x(\tau, \sigma) = e^{-iqy(\tau, \sigma)} X(\tau, \sigma) , \quad \partial_+ \partial_- X = 0 , \quad X = X_+(\sigma_+) + X_-(\sigma_-) , \quad (2.10)$$

$$X(\tau, \sigma + \pi) = e^{2\pi i \gamma} X(\tau, \sigma) , \quad \gamma \equiv qR w , \quad (2.11)$$

$$\lambda_{R,L}(\tau, \sigma) = e^{-iqy(\tau, \sigma)} \Lambda_{R,L}(\tau, \sigma) , \quad \partial_\pm \Lambda_{R,L} = 0 , \quad \Lambda_{R,L} = \Lambda_{R,L}(\sigma_\mp) , \quad (2.12)$$

$$\Lambda_{R,L}(\tau, \sigma + \pi) = \pm e^{2\pi i \gamma} \Lambda_{R,L}(\tau, \sigma) , \quad (2.13)$$

with the signs ‘ $\pm$ ’ in (2.13) corresponding to the Ramond (R) and Neveu-Schwarz (NS) sectors. The crucial observation is that if  $x, \lambda$  are on shell,  $y$  still satisfies the free-field equation:

$$\partial_+ \partial_- y = 0 , \quad y = y_* + y' , \quad y_* = y_0 + 2\alpha' p \tau + 2R w \sigma . \quad (2.14)$$

The explicit expressions for the fields  $X = X_+ + X_-$  and  $\Lambda_{L,R}$  are then

$$X_\pm(\sigma_\pm) = e^{\pm 2i\gamma \sigma_\pm} \mathcal{X}_\pm(\sigma_\pm) , \quad \mathcal{X}_\pm(\sigma_\pm \pm \pi) = \mathcal{X}_\pm(\sigma_\pm) , \quad (2.15)$$

$$\Lambda_{L,R}(\sigma_\pm) = e^{\pm 2i\gamma \sigma_\pm} \eta_{L,R}(\sigma_\pm) , \quad (2.16)$$

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<sup>7</sup> The twist parameter  $\gamma$  can be interpreted as a flux corresponding to the 2d field  $A_a = q \partial_a y$  on the cylinder,  $\int A = 2qRw \int d\sigma = 2\pi\gamma$ .

where  $\mathcal{X}_\pm$  and  $\eta_{L,R}$  are the free fields with the standard free closed string boundary conditions, i.e. having the standard oscillator expansions, e.g.,

$$\begin{aligned}\mathcal{X}_+ &= i \sum_{n \in \mathbf{Z}} \tilde{a}_n e^{-2in\sigma_+}, & \mathcal{X}_- &= i \sum_{n \in \mathbf{Z}} a_n e^{-2in\sigma_-}, \\ \eta_R^{(\text{NS})} &= \sum_{r \in \mathbf{Z} + \frac{1}{2}} c_r e^{-2ir\sigma_-}, & \eta_R^{(\text{R})} &= \sum_{n \in \mathbf{Z}} d_n e^{-2in\sigma_-},\end{aligned}\tag{2.17}$$

and similar expressions for the left fermions with oscillators having extra tildes. One can then proceed with the canonical quantization of the model expressing the observables in terms of the above free oscillators. It is convenient to choose the light-cone gauge, eliminating the oscillator part of  $u = y - t$  (see [3,17,4] for details). Then the string states are parametrised by the following global quantum numbers: the total energy  $E$ , the Kaluza-Klein linear momentum number  $m$  ( $p_y = mR^{-1}$ ) and the winding number  $w$ , the orbital momenta  $l_R$  and  $l_L$  in the 2-plane (analogues of the Landau level which replace the linear momenta  $p_1, p_2$ ) and by the continuous momentum  $p_3$  (as well as by discrete momenta corresponding to extra 5 toroidal directions).

The left and right angular momentum operators in the 2-plane contain the orbital momentum parts plus the spin parts

$$\hat{J}_{L,R} = \pm(l_{L,R} + \frac{1}{2}) + S_{L,R}, \quad \hat{J} \equiv \hat{J}_L + \hat{J}_R = l_L - l_R + S_L + S_R, \tag{2.18}$$

where the orbital momenta  $l_{L,R} = 0, 1, 2, \dots$  and  $S_{L,R}$  have the standard free superstring expressions [18] in terms of free oscillators, e.g.,  $S_R = \sum_{n=1}^{\infty} (b_{n+}^\dagger b_{n+} - b_{n-}^\dagger b_{n-})$  + fermionic terms ( $b_n$  are bosonic oscillators  $a_n$  in (2.17) rescaled by factors of  $(n \pm \gamma)^{1/2}$ ). In the case when  $\gamma = 0$  (or, more generally,  $\gamma = n$ ) the zero-mode structure changes in that the translational invariance in the 2-plane is restored (the zero mode oscillators are replaced by the standard zero-mode operators  $x_{1,2}, p_{1,2}$  [10,3]). The number of states operators  $\hat{N}_R$  and  $\hat{N}_L$  have the standard expressions [18] in terms of the oscillators in (2.17) so that  $\hat{N}_{R,L} = N_{R,L} - a$ ,  $a^{(\text{R})} = 0$ ,  $a^{(\text{NS})} = \frac{1}{2}$ ,  $N_R = \sum_{n=1}^{\infty} n(b_{n+}^\dagger b_{n+} + b_{n-}^\dagger b_{n-} + \dots)$ , etc. Under the usual GSO projection (which is implied by the GS formulation and is necessary for correspondence with the free RNS superstring theory in the limit  $q = 0$  but will not automatically lead to space-time supersymmetry for generic  $q$ )  $\hat{N}_R$  and  $\hat{N}_L$  can take only non-negative integer values. They satisfy the standard constraint  $\hat{N}_R - \hat{N}_L = mw$ .

Computing the stress tensor one finds the resulting expression for the light-cone gauge Hamiltonian<sup>8</sup>

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 - \alpha' q (Q_L \hat{J}_R + Q_R \hat{J}_L) + \frac{1}{2} \alpha' q^2 \hat{J}^2, \tag{2.19}$$

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<sup>8</sup> Here  $p_\alpha^2$  includes  $p_3^2$  as well as the contributions of the linear and winding momenta in other 5 free compactified dimensions (for simplicity we shall sometimes set them equal to zero).

$$\hat{\mathcal{H}}_0 \equiv \frac{1}{2}\alpha'(-E^2 + p_\alpha^2 + \frac{1}{2}Q_L^2 + \frac{1}{2}Q_R^2) + \hat{N}_R + \hat{N}_L, \quad Q_{L,R} = mR^{-1} \pm \alpha'^{-1}Rw.$$

It can be interpreted as describing charged states of closed superstring compactified on  $S^1$  moving in the Melvin flux tube background.  $\hat{\mathcal{H}}$  is different from the Hamiltonian of the free string on a circle  $\hat{\mathcal{H}}_0$  by  $O(q)$  ('gyromagnetic' interaction, cf. (1.1),(1.2)) and  $O(q^2)$  (charge-independent 'gravitational' interaction) terms. Charged string states are 'trapped' by the magnetic field (they cannot move freely in the 2-plane having discrete orbital momentum numbers  $l_L, l_R$  instead of continuous linear momenta  $p_1, p_2$ ).

$\hat{\mathcal{H}}$  can be represented also in the following ('free superstring compactified on a circle') form

$$\begin{aligned} \hat{\mathcal{H}} &= \frac{1}{2}\alpha'[-E^2 + p_\alpha^2 + (m - qR\hat{J})^2 R^{-2} + \alpha'^{-2}w^2 R^2] + \hat{N}_R + \hat{N}_L - qRw(\hat{J}_R - \hat{J}_L) \\ &= \frac{1}{2}\alpha'(-E^2 + p_\alpha^2 + m'^2 R^{-2} + \alpha'^{-2}w^2 R^2) + \hat{N}'_R + \hat{N}'_L, \end{aligned} \quad (2.20)$$

where<sup>9</sup>  $m' \equiv m - qR\hat{J}$ ,  $\hat{N}'_R \equiv \hat{N}_R - \gamma\hat{J}_R$ ,  $\hat{N}'_L \equiv \hat{N}_L + \gamma\hat{J}_L$ ,  $\gamma \equiv qRw$ . The Virasoro condition  $\hat{\mathcal{H}} = 0$  then leads to the expression for the mass spectrum  $M^2 \equiv E^2 - p_\alpha^2$ . The mass spectrum is invariant under

$$q \rightarrow q + 2nR^{-1}, \quad n = 0, \pm 1, \dots, \quad (2.21)$$

since (for  $w = 0$ ) this transformation can be compensated by  $m \rightarrow m - 2n\hat{J} = \text{integer}$ . Note that because  $\hat{J}$  can take both integer (NS-NS, R-R sectors) and *half-integer* (NS-R, R-NS sectors) values, the symmetry of the bosonic part of the spectrum  $q \rightarrow q + nR^{-1}$  is *not* a symmetry of its fermionic part, i.e. the full superstring spectrum is invariant only under (2.21).

The same conclusion about the periodicity in  $q$  is true in general for  $w \neq 0$ . In the form given above, eq. (2.19) is valid for  $0 \leq w < (qR)^{-1}$ , i.e. for  $0 \leq \gamma < 1$ . The generalization to other values of  $\gamma$  is straightforward [3,4]: one is to replace  $\gamma = qRw$  in (2.20) by  $\hat{\gamma} \equiv \gamma - [\gamma]$ , where  $[\gamma]$  denotes the integer part of  $\gamma$  ( $0 \leq \hat{\gamma} < 1$ , cf. (1.2)). For fixed radius  $R$  the mass spectrum is thus periodic in  $q$ , i.e. it is mapped into itself under (2.21) (combined with  $m \rightarrow m - 2n\hat{J}$ ). In the case of  $qR = 2n$  (i.e.  $\gamma = 2nw = 2k$ ) the spectrum is thus equivalent to that of the free superstring compactified on a circle. For  $qR = 2n + 1$  (i.e.  $\gamma = (2n + 1)w = 2k + 1$  if  $w$  is odd) the spectrum is the same as that of the free superstring compactified on a circle with antiperiodic boundary conditions for space-time fermions [19] (see also [20,21]). This relation will become clear in the GS formulation discussed below.<sup>10</sup>

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<sup>9</sup> Up to the orbital momentum terms,  $\hat{N}'_{R,L}$  can be put into the same form as free operators  $\hat{N}_{R,L}$  with the factor  $n$  replaced by  $n \pm \gamma$ . This is related to the fact that the model we are solving is 'locally trivial', i.e. the  $q$ -dependence could be eliminated by a rotation of coordinates if not for the global effects.

<sup>10</sup> In particular, it will be apparent that the interaction term in the superstring action can be eliminated by a globally defined field transformation only if  $qR = 2n$ , while for  $qR = 2n + 1$  this can be done at the expense of imposing antiperiodic boundary conditions (in  $\sigma$  or in the  $y$ -direction) on fermions (under the rotation by the angle  $2\pi qR = 2\pi$  in the 2-plane, which is associated with a periodic shift in  $y$ , the bosons remain invariant but spinors change sign).

### 2.3. Green-Schwarz formulation

Given a generic curved bosonic background, the corresponding Green-Schwarz (GS) superstring action [22] defines a complicated non-linear 2d theory. When one is able to fix a light-cone gauge and, moreover, the background geometry is flat as in the case of the Melvin model (2.3) (so that conformal invariance and  $\kappa$ -supersymmetry are guaranteed) the action becomes very simple (cf. (2.6))<sup>11</sup>

$$L_{GS} = G_{\mu\nu}(x)\partial_+x^\mu\partial_-x^\nu + i\mathcal{S}_R\mathcal{D}_+\mathcal{S}_R + i\mathcal{S}_L\mathcal{D}_-\mathcal{S}_L, \quad (2.22)$$

$$\mathcal{D}_a \equiv \partial_a + \frac{1}{4}\gamma_{mn}\omega_\mu^{mn}\partial_ax^\mu.$$

Here  $\mathcal{S}_{R,L}^p$  ( $p = 1, \dots, 8$ ) are the right and left real spinors of  $SO(8)$  (we consider type IIA theory). In the case of (2.3) we get (cf. (2.5))

$$\begin{aligned} L_{GS} = & (\partial_+ + iq\partial_+y)x(\partial_- - iq\partial_-y)x^* + \partial_+y\partial_-y \\ & + i\mathcal{S}_R(\partial_+ - \frac{1}{4}q\epsilon^{ij}\gamma_{ij}\partial_+y)\mathcal{S}_R + i\mathcal{S}_L(\partial_- - \frac{1}{4}q\epsilon^{ij}\gamma_{ij}\partial_-y)\mathcal{S}_L. \end{aligned} \quad (2.23)$$

It is natural to decompose the  $SO(8)$  spinors according to  $SO(8) \rightarrow SU(4) \times U(1)$ , i.e.  $\mathcal{S}_L^p \rightarrow (\mathcal{S}_L^r, \bar{\mathcal{S}}_L^r)$ ,  $\mathcal{S}_R^p \rightarrow (\mathcal{S}_R^r, \bar{\mathcal{S}}_R^r)$ ,  $r = 1, \dots, 4$ . Then the fermionic terms in (2.23) become

$$L_{GS}(\mathcal{S}) = i\bar{\mathcal{S}}_R^r(\partial_+ + \frac{1}{2}iq\partial_+y)\mathcal{S}_R^r + i\bar{\mathcal{S}}_L^r(\partial_- - \frac{1}{2}iq\partial_-y)\mathcal{S}_L^r. \quad (2.24)$$

The connection terms in the covariant derivatives in the fermionic part of the GS action (2.24) have extra coefficients  $\frac{1}{2}$  with respect to the ones in the RNS action (2.7). This immediately implies that the full theory is periodic under  $qR \rightarrow qR + 2n$ .

The condition that the GS action (2.22),(2.23) has residual supersymmetry invariance  $\mathcal{S} \rightarrow \mathcal{S} + \epsilon(x)$  is equivalent to  $\mathcal{D}_a\epsilon(x(\tau, \sigma)) = \partial_ax^\mu(\partial_\mu + \frac{1}{4}\gamma_{mn}\omega_\mu^{mn})\epsilon(x) = 0$ . The absence of supersymmetry invariance is the consequence of the absence of zero modes of the above covariant derivative operators, or, equivalently, of the non-existence of solutions of the Killing spinor equation

$$(\partial_\mu + \frac{1}{4}\gamma_{mn}\omega_\mu^{mn})\epsilon = 0. \quad (2.25)$$

In the  $D = 3$  background corresponding to (2.5)  $\epsilon = \epsilon(x^i, y)$  is a space-time spinor and  $\omega_\mu^{mn}$  is the same flat spin connection as in (2.6),(2.7) so that (2.25) reduces (after  $SU(4) \times U(1)$  split of  $\epsilon$  as in (2.24)) to

$$(\partial_y \mp \frac{1}{2}iq)\epsilon = 0. \quad (2.26)$$

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<sup>11</sup> Its form [23,24,25] can be explicitly determined, e.g., by comparing [24] with the known light-cone superstring vertex operators [18].

The formal solution of (2.26)  $\epsilon(y) = \exp(\pm \frac{1}{2}iq) \epsilon(0)$  does not, however, satisfy the periodic boundary condition in  $y$ ,  $\epsilon(y + 2\pi R) = \epsilon(y)$  (unless  $qR = 2n$  when the Killing spinor does exist, in agreement with the fact that in this case the theory is equivalent to the free superstring). The conclusion is that for  $qR \neq 2n$  there is no residual space-time supersymmetry in the higher-dimensional (e.g.,  $D = 5$  supergravity) counterpart of the  $a = \sqrt{3}$  Melvin background.<sup>12</sup>

As in the bosonic and RNS cases, one can explicitly solve the classical string equations corresponding to (2.23) (cf. (2.12),(2.13))

$$\mathcal{S}_{R,L}(\tau, \sigma) = e^{-\frac{i}{2}qy(\tau, \sigma)} \Sigma_{R,L}(\sigma_{\mp}) , \quad \Sigma_{R,L}(\tau, \sigma + \pi) = e^{i\pi\gamma} \Sigma_{R,L}(\tau, \sigma) , \quad (2.27)$$

with the final result that the only essential difference, as compared to the free superstring case, is the coupling of bosons and fermions to the zero-mode part of the flat  $U(1)$  connection  $\partial_a y_*$ . The expressions for the superstring Hamiltonian and mass spectrum are effectively the same as in the RNS approach (2.19).<sup>13</sup>

In general, the model with  $qR = 2n + 1$  ( $\gamma = 2k + 1$  for odd  $w$ ) is equivalent to the free superstring compactified on a twisted 3-torus (in the limit when the 2-torus part is replaced by 2-plane), or on a circle with antiperiodic boundary conditions for the fermions [19] (in particular, the theory with  $qR = 1$  and  $R < \sqrt{2\alpha'}$  will have tachyons).

The fundamental world-sheet fermions  $\mathcal{S}$  that appear in GS action (2.22) are always *periodic* in  $\sigma$  (this is necessary for supersymmetry of the model in the  $q = 0$  limit). This implies that the ‘redefined’ fermions  $\Sigma$  in (2.27) must change phase under a shift in  $y$ -direction. For  $qR = 2n + 1$  this results in antiperiodic boundary conditions for *space-time* fermions as functions of  $y$  (the space-time fields can be represented, e.g., as coefficients in expansion of a super string field  $\Phi(y, \mathcal{S}, \dots)$  in powers of world-sheet fermions). As a result, there exists a 1-parameter family of models interpolating between the standard supersymmetric  $qR = 0$  model with fermions which are periodic in  $y$  and a non-supersymmetric  $qR = 1$  model with fermions which are antiperiodic in  $y$ .

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<sup>12</sup> Even though the  $D = 5$  supergravity background  $M^5$  is flat, it is the presence of the flat but non-trivial spin connection that leads to the breaking of supersymmetry. Let us note also that the absence of Killing spinors in the case of the  $a = 0$  Melvin solution of the Einstein-Maxwell theory was pointed out in [12].

<sup>13</sup> For  $2k \leq \gamma < 2k + 1$  the operators  $\hat{N}_{L,R}$ ,  $\hat{J}_{L,R}$  have the usual free GS superstring form, which is similar to their form in the R-sector of the RNS formalism with vanishing zero-point energy. For  $2k - 1 \leq \gamma < 2k$  the operators  $\hat{N}_{L,R}$  have the ‘NS-sector’ form, i.e. they take half-integer eigenvalues starting from  $-\frac{1}{2}$ .

#### 2.4. Mass spectrum: supersymmetry breaking and (in)stability

The expression for the mass spectrum that follows from (2.19),(2.20) is

$$M^2 \equiv E^2 - p_\alpha^2 = M_0^2 - 2qR^{-1}m\hat{J} - 2\alpha'^{-1}qRw(\hat{J}_R - \hat{J}_L) + q^2\hat{J}^2 \quad (2.28)$$

$$= 2\alpha'^{-1}(\hat{N}_L + \hat{N}_R) + (m - qR\hat{J})^2R^{-2} + \alpha'^{-2}w^2R^2 - qRw(\hat{J}_R - \hat{J}_L) , \quad (2.29)$$

where  $M_0^2 = 2\alpha'^{-1}(\hat{N}_L + \hat{N}_R) + m^2R^{-2} + \alpha'^{-2}w^2R^2$  is the mass operator of the free superstring compactified on a circle. It is easy to see that in general  $M^2$  is not positive definite in the winding ( $w \neq 0$ ) sector because of the last  $O(qRw)$  gyromagnetic interaction term in (2.29). Thus one should expect the presence of instabilities, in agreement with the magnetic interpretation of the model and the existence of charged higher spin states in the spectrum.

Indeed, it follows from (2.28) that (i) the space-time supersymmetry is broken for  $qR \neq 2n$ , and (ii) there exists a range of values of parameters  $q$  and  $R$  for which there are tachyonic states in the spectrum.

The breaking of supersymmetry is of course expected in view of the magnetic interpretation of the model (the coupling is spin-dependent). Suppose that we start with the free superstring compactified on a circle  $y$  and study what happens with the spectrum when we switch on the magnetic field,  $q \neq 0$ . Since the mass shift in (2.28) involves *both* components  $\hat{J}_L$  and  $\hat{J}_R$  of the angular momentum the masses of bosons and fermions that were equal for  $q = 0$  will become different for  $q \neq 0$  (it is impossible to have both  $\hat{J}_L$  and  $\hat{J}_R$  equal for bosons and fermions). Supersymmetry is absent already in the non-winding sector (where the coupling is to the total angular momentum  $\hat{J}$ ).

For example, the free superstring massless ground states ( $\hat{N}_{L,R} = 0 = m = w$ ) will, according to (2.28), get masses  $M = |q\hat{J}|$  proportional to their total angular momenta, which must be integer for bosons and half-integer for fermions (cf. (2.18)). Note that these states are neutral, so that from the 4-dimensional point of view the shift in the masses can be interpreted as a gravitational effect. This shift implies, in particular, that supersymmetry is broken already at the field-theory ( $D = 5$  or  $D = 4$  supergravity) level, in agreement with the absence of Killing spinors in the  $D = 4$  Melvin background discussed above.

In the absence of supersymmetry some instabilities of the bosonic string model may survive also in the superstring case. The mass operator (2.29) is positive in the non-winding sector, but, as in the bosonic case, tachyonic states may appear in the winding sector. Consider, for example, the NS-NS winding states with zero Kaluza-Klein momentum and zero orbital momentum quantum numbers and with maximal absolute values of the spins  $S_{R,L}$  at given levels (leading Regge trajectory)

$$w > 0 , \quad m = 0 , \quad l_R = l_L = 0 , \quad S_R = \hat{N}_R + 1 , \quad S_L = -\hat{N}_L - 1 . \quad (2.30)$$

We shall assume that  $0 < qRw < 1$  (states with  $w > (qR)^{-1}$  can be analysed in a similar way). Then  $\hat{N}_R = \hat{N}_L \equiv N$ ,  $\hat{J} = 0$ ,  $\hat{J}_R - \hat{J}_L = 2N + 1$ , and

$$\alpha' M^2 = 4N + \alpha'^{-1} w^2 R^2 - 2qRw(2N + 1). \quad (2.31)$$

The state with given  $N$  and  $w$  will be tachyonic for  $q > q_{crit}$ ,  $q_{crit} = \frac{4N + \alpha'^{-1} w^2 R^2}{2(2N + 1)wR}$ . For  $N = 0$  we get  $\alpha' q_{crit} = \frac{1}{2}wR$ . The condition  $qRw < 1$  is satisfied provided  $wR < \sqrt{2\alpha'}$ .

In general, states with  $M^2 < 0$  can be present only for  $R < \sqrt{2\alpha'}$ , i.e. the full spectrum is *tachyon-free* if  $R > \sqrt{2\alpha'}$ . For fixed  $R < \sqrt{2\alpha'}$  the minimal value of the magnetic field strength parameter at which tachyons first appear is  $\alpha' q_{crit} = \frac{1}{2}R$ , corresponding to the  $N = 0, w = 1$  case discussed above.

All other sectors (R-R, R-NS, NS-R) are tachyon-free. The absence of tachyons in the fermionic sectors is a direct consequence of unitarity. Since a unitary tree-level  $S$ -matrix should correspond to a string field theory with a hermitian action, the ‘square’ of hermitian fermionic kinetic operator should be positive in any background. This translates into the positivity of  $M^2$  for the fermionic states in the case of static backgrounds. One implication is that similar models should have another general property of the spectrum: the states which become tachyonic should originate only from the states of the free superstring spectrum which belong to the leading Regge trajectory [4]. If there were bosonic tachyons not only on the leading Regge trajectory, but also on the subleading one, then a fermionic state with an ‘intermediate’ value of the spin (but otherwise the same quantum numbers) would have  $M^2 < 0$ . Since this is not allowed by unitarity, in any unitary superstring model corresponding to a static background tachyonic states can only appear on the first (bosonic) Regge trajectory. This is indeed true in the open superstring case (2.9) and in the present and more general closed superstring models discussed in Section 4 [4].

Let us note also that the fact that some higher spin winding states may become tachyonic means that there are new *massless* states at the critical values of the magnetic field. This suggests a possibility of symmetry enhancement in similar models. The magnetic perturbations may also reveal certain hidden symmetries of the superstring spectrum.<sup>14</sup>

One can consider also the heterotic version of the above model (where the magnetic field is embedded in the Kaluza-Klein sector) by combining the ‘left’ or ‘right’ part of the superstring model with the free internal part. Then [4] (cf. (2.29))

$$\alpha' M^2 = 2(\hat{N}_R + \hat{N}_L) + p_I^2 + \alpha'(mR^{-1} - q\hat{J})^2 + \alpha'^{-1} w^2 R^2 - 2qRw(\hat{J}_R - \hat{J}_L), \quad (2.32)$$

where  $\hat{N}_R - \hat{N}_L = mw + \frac{1}{2}p_I^2$ ,  $\hat{N}_R = 0, 1, 2, \dots$ ,  $\hat{N}_L = N_L - 1 = -1, 0, 1, \dots$ . In addition to the instabilities discussed above there are also new ones, which (for the ‘self-dual’ value of the radius  $R = \sqrt{\alpha'}$ ) appear for infinitesimal values of the magnetic field. These are the usual Yang-Mills-type magnetic instabilities, associated with the gauge bosons ( $m = w = \pm 1$ ,  $p_I^2 = l_R = l_L = 0$ ,  $\hat{N}_R = N_L = 0$ ,  $S_R = 1$ ,  $S_L = 0$ ) of the  $SU(2)_L$  group.

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<sup>14</sup> Let us note in this connection that a special symmetry of a general class of tachyon-free string models with finite 1-loop cosmological constant was discussed in [26].



## 2.5. Partition function

The basic properties of the spectrum are reflected in the 1-loop (torus) partition function  $Z$  of the model which will be non-vanishing for  $q \neq 2nR^{-1}$  due to the absence of the GS fermionic zero modes, i.e. the absence of supersymmetry.  $Z$  is straightforward to compute by computing the path integral in the GS formulation [4]. The first step is to expand  $y$  in eigen-values of the Laplacian on the 2-torus and redefine the fields  $x, x^*$  and  $\mathcal{S}_{L,R}, \bar{\mathcal{S}}_{L,R}$  in (2.23),(2.24) to eliminate the non-zero-mode part of  $y$  from the  $U(1)$  connection. The zero-mode part of  $y$  on the torus ( $ds^2 = |d\sigma_1 + \tau d\sigma_2|^2$ ,  $\tau = \tau_1 + i\tau_2$ ,  $0 < \sigma_a \leq 1$ ) is  $y_* = y_0 + 2\pi R(w\sigma_1 + w'\sigma_2)$ , where  $w, w'$  are integer winding numbers. Integrating over the fields  $x, x^*$  and  $\mathcal{S}_{L,R}^r, \bar{\mathcal{S}}_{L,R}^r$ , we get a ratio of determinants of scalar operators of the type  $\partial + iA$ ,  $\bar{\partial} - i\bar{A}$  with constant connection  $A = q\partial y_* = \pi\chi$ ,  $\chi \equiv qR(w' - \tau w)$ . The partition function has the simple form [4]

$$Z(R, q) = cV_7 R \int \frac{d^2\tau}{\tau_2^2} \sum_{w, w'=-\infty}^{\infty} \exp(-\pi(\alpha'\tau_2)^{-1}R^2|w' - \tau w|^2) \quad (2.33)$$

$$\times \mathcal{Z}_0(\tau, \bar{\tau}; \chi, \bar{\chi}) \frac{Y^4(\tau, \bar{\tau}; \frac{1}{2}\chi, \frac{1}{2}\bar{\chi})}{Y(\tau, \bar{\tau}; \chi, \bar{\chi})},$$

where

$$Y(\tau, \bar{\tau}; \chi, \bar{\chi}) = \exp\left[\frac{\pi(\chi - \bar{\chi})^2}{2\tau_2}\right] \left| \frac{\theta_1(\chi|\tau)}{\chi\theta_1'(0|\tau)} \right|^2. \quad (2.34)$$

The factor  $\mathcal{Z}_0$  in (2.33) stands for the contributions of the integrals over the constant fields  $x, x^*, \mathcal{S}_{L,R}, \bar{\mathcal{S}}_{L,R}$  which become zero modes in the free-theory ( $q = 0$ ) limit

$$\mathcal{Z}_0 = \frac{(\frac{1}{2}\chi\tau_2^{-1/2})^4 (\frac{1}{2}\bar{\chi}\tau_2^{-1/2})^4}{\chi\bar{\chi}\tau_2^{-1}} = 2^{-8}q^6 R^6 |w' - \tau w|^6 \tau_2^{-3}. \quad (2.35)$$

$\mathcal{Z}_0$  vanishes for  $q \rightarrow 0$  in agreement with the restoration of supersymmetry in this limit.<sup>15</sup>

The partition function vanishes at all supersymmetric points  $qR = 2n$  where the fermionic determinants have zero modes ( $\theta_1$ -functions in  $Y$ -factors in (2.33) have zeros for any  $w, w'$ ). More generally,  $Z$  is periodic in  $q$  (see (2.21))

$$Z(R, q) = Z(R, q + 2nR^{-1}), \quad n = 0, \pm 1, \dots \quad (2.36)$$

For  $qR = 2n + 1$  the partition function is the same as that of the free superstring compactified on a circle with antiperiodic boundary conditions for space-time fermions [19] (the

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<sup>15</sup> The  $q \rightarrow 0$  divergence of the bosonic ‘constant mode’ factor  $\sim q^{-2}$  corresponds to the restoration of the translational invariance in the  $x_1, x_2$ -plane in the zero magnetic field limit (this infrared divergence reproduces the factor of the area of the 2-plane).

dependence on odd  $qR$  can be eliminated from (2.24) at the expense of making  $\mathcal{S}_{R,L}, \bar{\mathcal{S}}_{R,L}$  to satisfy antiperiodic boundary conditions in  $\sigma$  or  $y$ , cf. (2.27)).

$Z$  is infrared-divergent for those values of the moduli  $q$  and  $R$  for which there are tachyonic states in the spectrum and is finite for all other values. In particular, it is finite for  $R > \sqrt{2\alpha'}$  and arbitrary  $q$ .  $Z$  is usually interpreted as a cosmological term or 1-loop effective potential for the moduli (so that one may study its extrema, etc., cf. [27]). Since (2.33) is non-negative and periodic in  $qR$ , its minima are at supersymmetric points. It is not clear, however, that this interpretation of  $Z$  as an effective potential applies in the present case of a space-time (i.e. not internal space) string model since the  $D = 4$  background is curved ( $Z$  is rather the full 1-loop effective action evaluated on a classical solution).

### 3. Superstring models for $a = 1$ Melvin and more general static magnetic flux tube backgrounds

In the previous section we have discussed the simplest possible static magnetic flux tube model. Now we shall consider more general models [3,4] corresponding to the 2-parameter flux tube backgrounds (1.4). It turns out that the superstring versions of these models (which depend on compactification radius, vector and axial magnetic field parameters  $R, q$  and  $\beta$ ) have properties analogous to those of the  $\beta = 0$  ( $a = \sqrt{3}$  Melvin) model. In particular, supersymmetry is broken for generic values of  $(\beta, q)$  and these models reduce to the free superstring theory when both  $qR$  and  $\alpha'\beta R^{-1}$  are even integers. The spectrum contains tachyonic states which (in agreement with the general remarks in Section 2.3) appear only in the bosonic NS-NS sector and belong to the leading Regge trajectory.

The string models corresponding to (1.4) with  $\beta > q$  are related to the models with  $\beta < q$  by the duality transformation in the Kaluza-Klein coordinate  $y$ .<sup>16</sup> More precisely, the  $(R, \beta, q)$  model is  $y$ -dual to  $(\alpha'R^{-1}, q, \beta)$  model so that the  $q = \beta$  ( $a = 1$  Melvin) is the ‘self-dual’ point. For fixed  $q$  these models thus fill the interval  $0 \leq \beta \leq q$  parametrized by  $\beta$ , with  $a = \sqrt{3}$  and  $a = 1$  Melvin models being the boundary points. The non-trivial  $\mathcal{M}^3 = (\rho, \varphi, y)$  part of the corresponding bosonic string Lagrangian is [3] (cf. (2.3))

$$L = \partial_+ \rho \partial_- \rho + F(\rho) \rho^2 (\partial_+ \varphi + q_+ \partial_+ y) (\partial_- \varphi + q_- \partial_- y) \quad (3.1)$$

$$+ \partial_+ y \partial_- y + \mathcal{R}[\phi_0 + \tfrac{1}{2} \ln F(\rho)] , \quad F^{-1} = 1 + \beta^2 \rho^2 , \quad q_{\pm} \equiv q \pm \beta .$$

Note that in addition to the metric of  $\mathcal{M}^3$  (which is no longer flat) there is also the antisymmetric tensor and dilaton backgrounds ( $\mathcal{R}$  is 2d curvature). This model is related

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<sup>16</sup> At the level of the effective action (1.3) this transformation is the usual  $T$ -duality map,  $\mathcal{A} \leftrightarrow \pm \mathcal{B}$ ,  $\sigma \rightarrow -\sigma$ ,  $\Phi \rightarrow \Phi$ ,  $\hat{G}_{\mu\nu} \rightarrow \hat{G}_{\mu\nu}$ ,  $\hat{H}_{\mu\nu\lambda} \rightarrow \hat{H}_{\mu\nu\lambda}$ , which is obviously a symmetry of (1.3).

to (2.3) by the formal  $O(2, 2; R)$  duality rotation (combination of a shift of  $\varphi$  by  $y$  and duality in  $y$ ). Indeed, it can be obtained from the model which is  $y$ -dual to (2.3) by first changing  $q \rightarrow \beta$ ,  $\tilde{y} \rightarrow y$  and then shifting  $\varphi \rightarrow \varphi + qy$ . This explains why this bosonic model is solvable [3,4] even though the 10-dimensional target space geometry is no longer flat. The equivalent form of (3.1) is

$$L = \partial_+ \rho \partial_- \rho + F(\rho)(\partial_+ y - \beta \rho^2 \partial_+ \varphi')(\partial_- y + \beta \rho^2 \partial_- \varphi') \quad (3.2)$$

$$+ \rho^2 \partial_+ \varphi' \partial_- \varphi' + \mathcal{R}[\phi_0 + \frac{1}{2} \ln F(\rho)] ,$$

where we have used the formal notation  $\varphi' = \varphi + qy$ . Introducing an auxiliary 2d vector field with components  $V_+, V_-$  we can represent (3.2) as follows, cf. (2.5) (this corresponds to ‘undoing’ the duality transformation mentioned above)

$$L = \frac{1}{2}(D_+ x \ D_-^* x^* + c.c.) + V_+ V_- - V_- \partial_+ y + V_+ \partial_- y , \quad (3.3)$$

$$D_\pm \equiv \partial_\pm + i\beta V_\pm + iq\partial_\pm y .$$

Now it is easy to understand why the classical equations of this model are explicitly solvable in terms of free fields and the partition function is computable. In spite of the  $y$ -dependence in the first term, the equation of motion for  $y$  still imposes the constraint that  $V_a$  has zero field strength,  $\mathcal{F}(V) = \partial_- V_+ - \partial_+ V_- = 0$ : the variation over  $y$  of the first term vanishes once one uses the equation for  $x$  (as follows from the fact that  $qy$ -terms can be formally absorbed into a phase of  $x$ ). Then  $V_+ = C_+ + \partial_+ \tilde{y}$ ,  $V_- = C_- + \partial_- \tilde{y}$ ,  $C_\pm = \text{const.}$  In the equations for  $V_+, V_-$  one can again ignore the variation of the first term in (3.3) since it vanishes under  $\mathcal{F}(V) = 0$ . We find that  $V_+ = C_+ + \partial_+ \tilde{y} = \partial_+ y$ ,  $V_- = C_- + \partial_- \tilde{y} = -\partial_- y$ . The solution of the model then effectively reduces to that of the model (2.3), the only extra non-trivial contribution being the zero-mode parts of the two dual fields  $y$  and  $\tilde{y}$ . Interchanging of  $q$  and  $\beta$  is essentially equivalent (after solving for  $C_+, C_-$ ) to interchanging of  $y$  and  $\tilde{y}$  and thus of momentum and winding modes. Eliminating  $C_+, C_-$  one gets terms quartic in the angular momentum operators in the final Hamiltonian. Similar approach applies to the computation of the partition function  $Z$ . Once  $x, x^*$  have been integrated out, the integrals over the constant parts of  $V_+, V_-$  cannot be easily computed for  $q\beta \neq 0$  and thus remain in the final expression [3.4].

This discussion has a straightforward generalization to superstring case. The corresponding RNS action now contains the quartic fermionic terms which reflect the presence of a non-trivial (generalized) curvature of the space  $\mathcal{M}^3$ . The direct analogue of the ‘first-order’ Lagrangian (3.3) is (cf. (2.7))<sup>17</sup>

$$L_{\text{RNS}} = \frac{1}{2}(D_+ x \ D_-^* x^* + c.c.) + \lambda_R^* D_+ \lambda_R + \lambda_L^* D_- \lambda_L \quad (3.4)$$

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<sup>17</sup> The fermionic part of this Lagrangian is reminiscent of the fermionic models studied in [28].

$$+ V_+ V_- - V_- \partial_+ y + V_+ \partial_- y .$$

The final expressions for the Hamiltonian and partition function then look very similar to the bosonic ones (the role of fermions is just to supersymmetrize the corresponding free superstring number of states and angular momentum operators and to cancel certain normal ordering terms). One finds (cf. (2.19))

$$\begin{aligned} \hat{\mathcal{H}} = & \frac{1}{2}\alpha'(-E^2 + p_\alpha^2) + \hat{N}_R + \hat{N}_L \\ & + \frac{1}{2}\alpha'R^{-2}(m - qR\hat{J})^2 + \frac{1}{2}\alpha'^{-1}R^2(w - \alpha'\beta R^{-1}\hat{J})^2 - \hat{\gamma}(\hat{J}_R - \hat{J}_L) , \end{aligned} \quad (3.5)$$

where  $\hat{N}_R - \hat{N}_L = mw$ ,  $\hat{\gamma} \equiv \gamma - [\gamma]$ ,  $\gamma \equiv qRw + \alpha'\beta R^{-1}m - \alpha'q\beta\hat{J}$ .  $[\gamma]$  denotes the integer part of  $\gamma$  and the operators  $\hat{N}_{R,L}$ ,  $\hat{J}_{R,L}$  are the same as in (2.19).

The duality symmetry in the compact Kaluza-Klein direction  $y$  (which interchanges the axial and vector magnetic field parameters  $\beta$  and  $q$ ) is now manifest: (3.5) is invariant under  $R \leftrightarrow \alpha'R^{-1}$ ,  $\beta \leftrightarrow q$ ,  $m \leftrightarrow w$ . The resulting expression for the mass spectrum can be written in terms of the ‘left’ and ‘right’ magnetic field parameters and charges,  $q_\pm \equiv q \pm \beta$ ,  $Q_{L,R} = mR^{-1} \pm \alpha'^{-1}Rw$  (it reduces to (2.28) when  $\beta = 0$ )

$$M^2 = M_0^2 - 2(q_+Q_L\hat{J}_R + q_-Q_R\hat{J}_L) + (q_+^2\hat{J}_R + q_-^2\hat{J}_L)\hat{J} . \quad (3.6)$$

The only states which can be tachyonic are bosonic states on the first Regge trajectory with the maximal value for  $S_R$ , minimal value for  $S_L$ , and zero orbital momentum, i.e.  $\hat{J}_R = S_R - \frac{1}{2} = \hat{N}_R + \frac{1}{2}$ ,  $\hat{J}_L = S_L + \frac{1}{2} = -\hat{N}_L - \frac{1}{2}$ . Then

$$\alpha'M^2 = 2(\hat{N}_R + \hat{N}_L)(1 - \hat{\gamma}) + \alpha'R^{-2}(m - qR\hat{J})^2 + \alpha'^{-1}R^2(w - \alpha'\beta R^{-1}\hat{J})^2 - 2\hat{\gamma} , \quad (3.7)$$

which is not positive definite due to the last term  $-2\hat{\gamma}$ . One finds [4] that for generic values of  $(q, \beta)$  there are instabilities (associated with states with high spin and charge) for arbitrarily small values of the magnetic field parameters. The special case of  $\beta = 0$  (or  $q = 0$ ), corresponding to the  $a = \sqrt{3}$  Melvin model discussed in Section 2, is the only exception: in this (type II) model there are no tachyons below some *finite* value of  $q$ . The example which illustrates the generic pattern is the  $a = 1$  Melvin model where  $q = \beta$  ( $q_- = 0, q_+ = 2\beta$ ) and

$$\alpha'M^2 = 4\hat{N}_R + \alpha'Q_R^2 - 4\hat{\gamma}\hat{J}_R , \quad \gamma = \alpha'\beta Q_L - \alpha'\beta^2\hat{J} . \quad (3.8)$$

If we choose for  $R = \sqrt{\alpha'}$  then the states with  $w = m$ ,  $\hat{N}_L = 0$ ,  $\hat{J}_R = \hat{N}_R + \frac{1}{2}$  and  $\hat{J}_L = -\frac{1}{2}$  become tachyonic for  $\beta$  in the interval  $\beta_1 < \beta < \beta_2$ ,  $\beta_{1,2} = m^{-1}(1 \mp \sqrt{1 - \gamma_{crit}})$ ,  $\gamma_{crit} = m^2/(m^2 + \frac{1}{2})$ . For large  $m = \hat{N}_R$  these magnetic field parameters are very small. These infinitesimal instabilities appear because of the presence of states with arbitrarily large

charges and thus are a special property of the *closed* string theory. Unlike the usual Yang-Mills-type magnetic instabilities, they (being associated with higher level states) remain even after the massless-level states get small masses. This suggests that a configuration with generic values of  $(q, \beta)$  will ‘decay’ to become a stable one with special values of  $q, \beta$ .

The supersymmetry is broken for generic values of the magnetic field parameters  $\beta, q$  (the two magnetic fields couple to both  $L, R$ - components of the spin which cannot simultaneously be the same for bosons and fermions). When  $qR = 2n_1$  and  $\alpha'\beta R^{-1} = 2n_2$ ,  $n_{1,2} = 0, \pm 1, \dots$ , the theory is equivalent to the free superstring compactified on a circle (in this case  $\hat{\gamma} = 0$  and, after appropriate shifts of  $m, w$  by integers, (3.5) reduces to the free superstring Hamiltonian). If  $qR = 2n_1 + 1$  or  $\alpha'\beta R^{-1} = 2n_2 + 1$ , then the necessary shift in  $m$  or  $w$  in the fermionic sector involves half-integer numbers. In these cases the theory can be interpreted as a free superstring on a circle with antiperiodic boundary conditions for space-time fermions.

The corresponding partition function is [3,4] (cf. (2.33))

$$\begin{aligned}
Z(R, q, \beta) = & cV_7 R \int \frac{d^2\tau}{\tau_2^2} \int dC d\bar{C} (\alpha'\tau_2)^{-1} \sum_{w, w'=-\infty}^{\infty} \\
& \times \exp \left( -\pi(\alpha'\beta^2\tau_2)^{-1} [\chi\bar{\chi} - R(q+\beta)(w' - \tau w)\bar{\chi} - R(q-\beta)(w' - \bar{\tau}w)\chi \right. \\
& \left. + R^2 q^2 (w' - \tau w)(w' - \bar{\tau}w)] \right) \times \mathcal{Z}_0(\tau, \bar{\tau}; \chi, \bar{\chi}) \frac{Y^4(\tau, \bar{\tau}; \frac{1}{2}\chi, \frac{1}{2}\bar{\chi})}{Y(\tau, \bar{\tau}; \chi, \bar{\chi})},
\end{aligned} \tag{3.9}$$

where  $\chi \equiv 2\beta C + qR(w' - \tau w)$ ,  $\bar{\chi} \equiv 2\beta\bar{C} + qR(w' - \bar{\tau}w)$ , and  $Y(\tau, \bar{\tau}; \chi, \bar{\chi})$  and  $\mathcal{Z}_0(\tau, \bar{\tau}; \chi, \bar{\chi})$  were defined in (2.34) and (2.35). The auxiliary parameters  $C, \bar{C}$  are proportional to the constant parts of  $V_{\pm}$  in (3.4). In the limit  $\beta \rightarrow 0$  we recover the partition function (2.33) of the model discussed in the previous section.

The partition function (3.9) has the following symmetries (cf. (2.36))

$$Z(R, q, \beta) = Z(\alpha'R^{-1}, \beta, q), \quad Z(R, q, \beta) = Z(R, q + 2n_1 R^{-1}, \beta + 2n_2 \alpha'^{-1} R). \tag{3.10}$$

These are symmetries of the full conformal field theory (as can be seen directly from the string action in the Green-Schwarz formulation). If  $qR \neq n_1$  and  $\alpha'\beta R^{-1} \neq n_2$ , there are tachyons for any value of the radius  $R$ , and the partition function contains infrared divergences. As follows from (3.10), when  $\alpha'\beta R^{-1}$  (or  $qR$ ) is an even integer, the partition function reduces to that of the  $a = \sqrt{3}$  Melvin model (2.33). In particular, in the special case when both  $qR$  and  $\alpha'\beta R^{-1}$  are even, the partition function is identically zero (then the theory is equivalent to the free superstring). When either  $\alpha'\beta/R$  or  $qR$  is an odd integer, the partition function is finite in a certain range of values of the radius.

#### 4. ‘Twisted’ $SU(2) \times U(1)$ WZNW model as compact analogue of magnetic flux tube model and supersymmetry breaking

The non-trivial part of the 10-dimensional space-time corresponding to the  $a = \sqrt{3}$  Melvin model (2.3),(2.7) is a flat non-compact space  $\mathcal{M}^3$ . The breaking of supersymmetry in the model (2.7),(2.22) is a consequence of an incompatibility between periodicity of space-time spinors in the Kaluza-Klein direction  $y$  and the presence of mixing between  $y$  and the angular coordinate of 2-plane which produces a flat but globally non-trivial spin connection. Replacing the 2-plane by a *compact* space with a non-trivial isometry and mixing the isometric coordinate with another compact internal coordinate  $y$ , one may try to construct similar models in which supersymmetry is broken while the Lorentz symmetry in the remaining flat non-compact directions is preserved. Below we shall discuss such a model where the 4-space  $R_{x_3} \times \mathcal{M}^3 = (x_3, \rho, \varphi, y)$  is replaced by  $\mathcal{M}^4 = SU(2) * U(1)$ , i.e. the ‘twisted’ product of the  $SU(2)$  WZNW model and a circle.  $\mathcal{M}^4$  (plus the corresponding torsion) is locally the group space  $SU(2) \times U(1)$  and thus defines a conformal model.

We shall consider  $\mathcal{M}^4$  as (part of) the *internal* space and discuss how the twist parameters lead to supersymmetry breaking.<sup>18</sup> This will not be in contradiction with the ‘no-go’ theorem on impossibility of spontaneous supersymmetry breaking by continuous parameters [30,31] since the corresponding (heterotic) string vacuum will not be supersymmetric already in the absence of the twist (the central charge condition will not be satisfied unless one adds a linear dilaton background or considers special values of the level  $k$ ). In what follows we shall ignore this problem, concentrating on ‘additional’ supersymmetry breaking induced by continuous twist parameters. In view of a relation to the magnetic flux tube model, this supersymmetry breaking can be given a ‘magnetic’ interpretation.<sup>19</sup>

Simplest examples of models with spontaneous supersymmetry breaking are string compactifications on ‘twisted’ tori (or string analogues of the ‘Scherk-Schwarz’ [33] compactifications) [19,34,21]. Consider, e.g., the 3-torus  $(x_1, x_2, y) \equiv (x_1 + 2\pi R' n_1, x_2 + 2\pi R' n_2, y + 2\pi R m)$  and twist it by imposing the condition that the shift by period in  $y$  should be accompanied by a rotation in the  $(x_1, x_2)$ -plane. For a *finite*  $R'$  the only possible rotations are by angles  $\frac{1}{2}\pi m$ , i.e. one may identify the points  $(\theta, y) = (\theta + 2\pi n + \frac{1}{2}\pi m, y + 2\pi R m)$ ,  $\cot \theta = x_1/x_2$ . The superstring theory with this flat but non-trivial 3-space  $\mathcal{M}_0^3$  as part of the internal space was considered in [19] (see also [34,21]) where it was found that such twist of the torus breaks supersymmetry and leads to the

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<sup>18</sup> A special case of this model was considered in [29] where it was interpreted as describing a space-time magnetic background (with the curvature of  $S^3$  being small, i.e. the level of  $SU(2)$  being large).

<sup>19</sup> In that sense this model is similar to other models where supersymmetry is broken (in a discrete way) by magnetic fields in internal dimensions, see [32] and refs. there.

existence of tachyons for  $R^2 < 2\alpha'$  and non-vanishing (and finite for  $R^2 > 2\alpha'$ ) partition function. The  $R' \rightarrow \infty$  limit of this model is actually equivalent to the special case  $qR = \frac{1}{4}m$  of the  $a = \sqrt{3}$  Melvin model (2.5) (the case of  $m = 4$  explicitly considered in [19] is equivalent to the superstring compactified on a circle with antiperiodic boundary conditions for the fermions). Since in the model (2.5) the 2-plane is non-compact and thus the twisting angle  $2\pi qR$  is arbitrary, this model continuously connects large  $R'$  limits of the models of [19] with different values of the integer  $m$ .

Such models with compact *flat* internal spaces always have *discrete* allowed values of the twisting parameter (a symmetry group of a lattice which generates a torus from  $R^N$  is discrete). As a result, the supersymmetry breaking mass scale  $\mu$  is directly proportional to the compactification mass scale  $R'^{-1}$  [35]. Given that  $\mu$  should be of *TeV* order, this implies the large value for  $R'$  and thus the existence of a tower of ‘light’ ( $M \sim \text{TeV}$ ) Kaluza-Klein states [36,37]. This leads to fast growth of coupling with energy making perturbation theory unapplicable [36].

It could happen that analogous ‘twistings’ of models with compact *curved* internal spaces with isometries lead to vacua where the supersymmetry breaking scale could be continuously adjusted and thus decoupled from the Kaluza-Klein scale. Such possibility, however, is ruled out by the results of [30,31].<sup>20</sup> Even though this will not resolve the problem of large internal dimension (since the ‘discrete’ part of the supersymmetry breaking will already relate the supersymmetry breaking and compactification scales) it may still be of interest to look for other continuous mechanisms of supersymmetry breaking which may complement the ‘discrete’ one. The idea is to separate the issue of discrete supersymmetry breaking due to non-compensation of the central charge from the *additional* continuous supersymmetry breaking induced by the twists.

To construct a generalisation of the model (2.2),(2.3) which will be compact and, at the same time, remain to be conformal, let us consider the  $SU(2) \times U(1)$  WZNW theory and ‘twist’ the product by shifting the two isometric Euler angles  $\theta_L$  and  $\theta_R$  of  $SU(2)$  ( $g = \exp(\frac{i}{2}\theta_L\sigma_3)\exp(\frac{i}{2}\psi\sigma_2)\exp(\frac{i}{2}\theta_R\sigma_3)$ ) by the periodic coordinate  $y \in (0, 2\pi R)$  corresponding to  $U(1)$

$$\theta'_L = \theta_L + q_1 y, \quad \theta'_R = \theta_R + q_2 y. \quad (4.1)$$

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<sup>20</sup> According to [30] it is not possible to break supersymmetry in a continuous way by adding a marginal perturbation to an  $N = 2$  supersymmetric world sheet conformal model describing a space-time supersymmetric vacuum of heterotic string theory. This ‘no-go’ theorem does not apply to the non-compact model (2.3),(2.7) which can be considered as a marginal perturbation of the free non-compact model: the left-right symmetric perturbation  $q\epsilon_{ij}(x_i\partial x_j\bar{\partial}y + x_i\bar{\partial}x_j\partial y)$  in (2.5) is marginal and integrable (since the space  $\mathcal{M}^3$  is flat so that  $R_{\mu\nu} = 0$  order by order in  $q$ ) but is not well-defined as a CFT operator. Note that according to (2.19),(2.28) the supersymmetry is broken there already at  $O(q)$  order.

Here  $q_1, q_2$  are continuous twist parameters.<sup>21</sup> The special case of such model with  $q_2 = 0$  was considered in [29]. To make contact with the model (2.3) it is necessary to keep both  $q_1$  and  $q_2$  non-vanishing [4].

The resulting  $SU(2) \times U(1)$  WZNW Lagrangian

$$L'(q_1, q_2) = L_{SU(2)}(\psi, \theta'_L, \theta'_R) + \partial y \bar{\partial} y, \quad (4.2)$$

$$L_{SU(2)}(\psi, \theta_L, \theta_R) = k(\partial\psi\bar{\partial}\psi + \partial\theta_L\bar{\partial}\theta_L + \partial\theta_R\bar{\partial}\theta_R + 2\cos\psi\partial\theta_R\bar{\partial}\theta_L), \quad (4.3)$$

defines a conformal theory since locally  $\mathcal{M}^4 = (\psi, \theta_L, \theta_R, y)$  is still the same  $SU(2) \times U(1)$  group manifold. In particular, the corresponding central charge is unchanged,  $c = \frac{3k}{k+2} + 1$ . Its independence of  $q_i$  makes it clear that ‘trivial’ discrete breaking of supersymmetry due to non-zero central charge deficit will be unrelated to ‘non-trivial’ continuous one induced by non-vanishing ‘magnetic’ twists  $q_i$ .

The case of  $q_1 = -q_2 = q$  is a compact analogue of the model (2.3). Let us first note that  $L_{SU(2)}$  in (4.3) can be written as

$$\begin{aligned} L_{SU(2)}(\psi, \theta, \tilde{\theta}) &= k\left[\partial\psi\bar{\partial}\psi + 4\sin^2\frac{\psi}{2}\partial\theta\bar{\partial}\theta + 4\cos^2\frac{\psi}{2}\partial\tilde{\theta}\bar{\partial}\tilde{\theta} \right. \\ &\quad \left. + 2\cos\psi(\partial\tilde{\theta}\bar{\partial}\theta - \partial\theta\bar{\partial}\tilde{\theta})\right], \quad \theta = \frac{1}{2}(\theta_L - \theta_R), \quad \tilde{\theta} = \frac{1}{2}(\theta_L + \theta_R). \end{aligned} \quad (4.4)$$

For small  $\psi$  (large  $k$ ) (4.4) reduces to  $k(\partial\psi\bar{\partial}\psi + \psi^2\partial\theta\bar{\partial}\theta + 4\partial\tilde{\theta}\bar{\partial}\tilde{\theta} + \dots)$ , i.e. describes a product of a 2-disc  $(\psi, \theta)$  and a line  $\tilde{\theta}$ . Observing that for  $q_1 = -q_2 = q$  the shift (4.1) implies  $\theta' = \theta + qy$ ,  $\tilde{\theta}' = \tilde{\theta}$ , we can establish a relation between  $L'(q, -q)$  (4.2) and (2.3) by identifying the coordinates in the following way:  $\sqrt{k}\psi \rightarrow \rho$ ,  $\theta \rightarrow \varphi$ ,  $\sqrt{k}\tilde{\theta} \rightarrow x_3$ .

The Lagrangian (4.2) can be represented in the form of a perturbation of the  $SU(2) \times U(1)$  WZNW model

$$\begin{aligned} L'(q_1, q_2) &= L_{SU(2)}(\theta_L, \theta_R, \psi) + 2q_1\bar{J}_3J_y + 2q_2J_3\bar{J}_y \\ &\quad + (1 + kq_1^2 + kq_2^2 + 2kq_1q_2\cos\psi)\partial y\bar{\partial} y, \end{aligned} \quad (4.5)$$

where  $J_3 = -ik\text{Tr}(\sigma_3 g^{-1}\partial g)$ ,  $\bar{J}_3 = -ik\text{Tr}(\sigma_3 \bar{\partial} g g^{-1})$ ,  $J_y, \bar{J}_y$  are the Cartan currents of the  $SU(2) \times U(1)$  model<sup>22</sup>

$$J_3 = k(\partial\theta_L + \cos\psi\partial\theta_R), \quad \bar{J}_3 = k(\bar{\partial}\theta_R + \cos\psi\bar{\partial}\theta_L), \quad J_y = \partial y, \quad \bar{J}_y = \bar{\partial} y. \quad (4.6)$$

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<sup>21</sup> The discussion that follows can be generalised to the case of other WZNW models (and their cosets) using the parametrisation  $g = \exp(i\theta_L^s H_s) \exp(i\psi^\alpha E_\alpha) \exp(i\theta_R^s H_s)$  (where  $H_s$  are generators of the Cartan subalgebra) and mixing  $\theta_L, \theta_R$  with coordinates of an extra torus.

<sup>22</sup> Another way to see why (4.2), (4.5) with  $q_1 = -q_2 = q$  is related to (2.3), (2.5) is to use the parametrization  $g = \exp(\frac{i}{2\sqrt{k}}x_n\sigma_n)$  in which (for small  $x_n$  or large  $k$ )  $J_3 = \sqrt{k}\partial x_3 + \epsilon_{ij}x_i\partial x_j + \dots$ ,  $\bar{J}_3 = \sqrt{k}\bar{\partial} x_3 - \epsilon_{ij}x_i\bar{\partial} x_j + \dots$ ,  $L_{SU(2)} = \partial x_i\bar{\partial} x_i + \partial x_3\bar{\partial} x_3 + \dots$ ,  $i, j = 1, 2$ . Then  $O(q)$  terms in (4.5) coincide with  $O(q)$  terms in  $L_1 + \partial x_3\bar{\partial} x_3$  in (2.5).



$O(q_1)$  and  $O(q_2)$  terms in (4.5) are thus integrable marginal perturbations of the  $SU(2) \times U(1)$  WZNW model. This follows directly from conformal invariance of (4.2) and is also in agreement with the fact that marginal  $J\bar{J}$ -perturbations by Cartan currents are integrable [38]. Note that it is only in the ‘chiral’ case of  $q_1 = 0$  or  $q_2 = 0$  considered in [29] that  $L'$  (4.2) can be represented as the original  $SU(2) \times U(1)$  CFT (with rescaled radius of  $y$ ) plus  $J\bar{J}$ -term.

The  $SU(2) \times U(1)$  group space preserves half of maximal space-time supersymmetry ( $N = 4, D = 4$ ); in particular, there is the corresponding number of Killing spinors. The superstring compactification on  $S^3 \times S^1$  was discussed, e.g., in [25] where it was pointed out that in the context of the effective field theory approach the extra condition [23] on the Killing spinor  $\gamma^{mnk}\hat{H}_{mnk} = 0$  coming from the dilatino transformation law (under the assumption that the dilaton is constant) is not satisfied. This is related to the issue of cancellation of the central charge, i.e. the presence or absence of the tree-level potential term for the dilaton (‘cosmological constant’). For special values of  $k$  the central charge can be cancelled by combining this model with a minimal model [39] or with ‘untwisted’  $N = 2$  coset model [40]. A central charge deficit can be compensated by a linear dilaton background with resulting ‘discrete’ supersymmetry breaking [36].<sup>23</sup> As mentioned above, we shall concentrate on additional supersymmetry breaking induced by the continuous twist parameters  $q_i$ .

Let us show that the background associated with the twisted model (4.2) does not admit Killing spinors so that the corresponding light-cone gauge Green-Schwarz superstring action does not have residual supersymmetry. In the case of the WZNW model the fermionic part of GS action is given by [24,25] (cf. (2.22))

$$L_{GS}(\mathcal{S}) = i\mathcal{S}_R\mathcal{D}_+\mathcal{S}_R + i\mathcal{S}_L\mathcal{D}_-\mathcal{S}_L, \quad (4.7)$$

$$\mathcal{D}_\pm \equiv \partial_\pm + \frac{1}{4}\gamma_{mn}\omega_{\pm\mu}^{mn}\partial_\pm x^\mu, \quad \omega_{\pm\mu}^{mn} = \omega_\mu^{mn} \pm \frac{1}{2}H^{mn}{}_\mu.$$

The quartic fermionic terms are absent since the generalised curvature  $R(\omega_\pm)$  vanishes.<sup>24</sup> If one chooses the left-invariant vierbein basis ( $e^m = e_\mu^m dx^\mu = -i\text{Tr}(\sigma^m g^{-1} dg)$ ) then one of the two generalised connections vanishes ( $H_{\mu\nu\lambda} = -f_{mnk}e_\mu^m e_\nu^n e_\lambda^k$ ):  $\omega_{-\mu}^{mn} = 0$ ,  $\omega_{+\mu}^{mn} = -\frac{3}{2}f_{\mu}^{mn}$  (in the right-invariant basis  $\omega_{+\mu}^{mn} = 0$ ). As a result, half of the maximal supersymmetry is preserved since  $\mathcal{S}_R$  fermions remain free (the corresponding Killing spinor equation (2.25) with  $\omega \rightarrow \omega_+$  has  $\epsilon = \text{const}$  as a solution). It may seem that the same

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<sup>23</sup> For a discussion of supersymmetric  $R_Q \times SU(2)_k$ -type models with linear dilaton see [41,29].

<sup>24</sup> The corresponding supersymmetric  $\sigma$ -model (RNS) action was discussed in [42,43]. Let us note that the twist (4.1) *preserves* the extended world-sheet supersymmetry the  $\sigma$ -model action since the existence of the supersymmetry is determined by local conditions on a background.

conclusion should be true also in the ‘shifted’ theory (4.2) since locally (4.1) can be considered as a coordinate transformation and thus the transformed  $\omega_{-\mu}^{mn}$  should also vanish. However, the left-invariant vierbein basis  $e_\mu^m$  which is independent of  $\theta_L$  explicitly depends on  $\cos\theta_R$ ,  $\sin\theta_R$  and thus its direct analogue obtained by making the shift (4.1) is not defined unless  $q_i R$  are integers. If one uses the original left-invariant basis, one needs to make an extra local Lorentz transformation to make the full metric diagonal. Then  $\mathcal{D}_-$  gets a flat but non-trivial  $q_i$ -dependent connection term. As a result, for generic  $q_i R \neq 2n_i$  the Killing spinor equation  $\mathcal{D}_{-\mu}\epsilon = 0$  has no solutions consistent with periodic boundary conditions in  $y$  (cf. (2.23),(2.24),(2.26)), i.e. supersymmetry is completely broken.

This can be seen more explicitly by choosing another (‘isometry-adapted’) basis corresponding to the diagonal form of the WZNW action (4.4):  $e^1 = d\psi$ ,  $e^2 = 2\sin\frac{\psi}{2}d\theta$ ,  $e^3 = 2\cos\frac{\psi}{2}d\tilde{\theta}$ ,  $e^y = dy$ . Then  $\omega^{\theta\psi} = -\cos\frac{\psi}{2}d\theta$ ,  $\omega^{\tilde{\theta}\psi} = \sin\frac{\psi}{2}d\theta$ ,  $\omega^{\theta\tilde{\theta}} = 0$ ,  $H = \frac{1}{2}e^\psi \wedge e^\theta \wedge e^{\tilde{\theta}}$ ,  $\omega_\pm^{\theta\psi} = -\cos\frac{\psi}{2}d(\theta \pm \tilde{\theta})$ ,  $\omega_\pm^{\tilde{\theta}\psi} = \sin\frac{\psi}{2}d(\tilde{\theta} \pm \theta)$ . The solution of the corresponding Killing equations is  $\epsilon_{R,L}(\psi, \theta, \tilde{\theta}) = \exp(\frac{i}{2}\psi\sigma_3)\exp(\pm\frac{i}{2}\theta_{R,L}\sigma_1)\epsilon_{R,L}(0)$ , where  $\theta_{L,R} = \tilde{\theta} \pm \theta$  and  $\epsilon_{L,R}(0)=\text{const.}$  Making the transformation (4.1) we find that the formal solutions of the Killing spinor equations corresponding to the ‘twisted’ model are (cf. (2.24),(2.26)):

$$\epsilon_L = e^{\frac{i}{2}\psi\sigma_3} e^{-\frac{i}{2}\theta_L\sigma_1} e^{-\frac{i}{2}q_1 y\sigma_1} \epsilon_L(0) , \quad \epsilon_R = e^{\frac{i}{2}\psi\sigma_3} e^{\frac{i}{2}\theta_R\sigma_1} e^{\frac{i}{2}q_2 y\sigma_1} \epsilon_L(0) . \quad (4.8)$$

As in (2.26) the periodic boundary conditions in  $y$  imply that supersymmetry is broken unless  $q_i R = 2n_i$ .

The dependence of the Killing spinors on  $\theta_L$  and  $\theta_R$  is related to the existence of the fixed points ( $\psi = 0, 2\pi$ ) in the action of the isometries corresponding to shifts along  $\theta_L$  and  $\theta_R$ . The same is true in the 2-plane case in the polar coordinate basis. The breaking of supersymmetry by the twist (4.1) may be attributed to this dependence. Similar breaking will thus happen in general when an isometry which has fixed points is ‘mixed’ with another circular dimension. This dependence of Killing spinors on angular coordinates leads also to an apparent ‘breakdown’ of supersymmetry [44] after the duality transformations in these coordinates (it is only the local realisation of supersymmetry that is actually broken by the duality [45,46]). Note that the shift (4.1) becomes part of the  $O(3, 3; Z)$  duality transformation group of the  $SU(2) \times U(1)$  model [47] only when  $q_i R = 2n_i$ , i.e. when supersymmetry is unbroken.

It would be interesting to determine the spectrum of the model (4.2) to see the supersymmetry breaking explicitly. This can probably be done by generalizing the approach of [29] where the special case of  $q_2 = 0$  was solved. It is clear from the spectrum given in [29] that  $q_1$  plays the role of the supersymmetry breaking parameter.

To try to construct ‘realistic’ models which include this ‘magnetic’ supersymmetry breaking one needs to address the question of saturation of the central charge condition. One possible suggestion is to relax this condition, assuming that the dilaton equation

should eventually be satisfied with loop and non-perturbative corrections included [36,32] (note, however, that  $\delta c \sim 1/k$  is small only if  $k$  is large and that leads back to the problem of large compactification scale). Another is to consider special values of  $k$  for which the total  $c$  can be balanced by combining this model, e.g., with  $N = 2$  coset one. An interesting aspect of the supersymmetry breaking induced by the ‘magnetic’ twists is that the resulting contribution to the cosmological constant is likely to be very small. Assuming that the compactification radius is  $R \sim \sqrt{\alpha'} \sim M_{Pl}^{-1}$  while the ‘magnetic’ supersymmetry breaking scale  $\mu^2 \sim q$  is of order of  $TeV^2$  or  $M_W^2$ , and that the partition function of a ‘realistic’ model will be similar to (2.33) (which is proportional to  $q^6$  for small  $q$ ), one may expect to find  $\Lambda_4 \sim M_W^{12}/M_{Pl}^8$  which is very small indeed.

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